# There is a Bluebird in my Talk that Wants to Get out

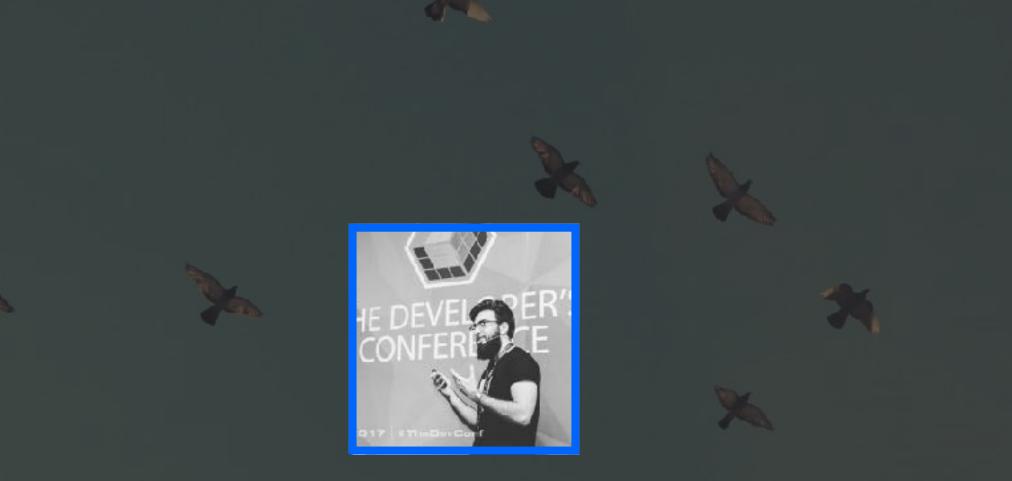
**Rewriting JavaScript** 





THEWIZARDLUCAS





# I'm Lucas, The Wizard





## lucasfcosta.com





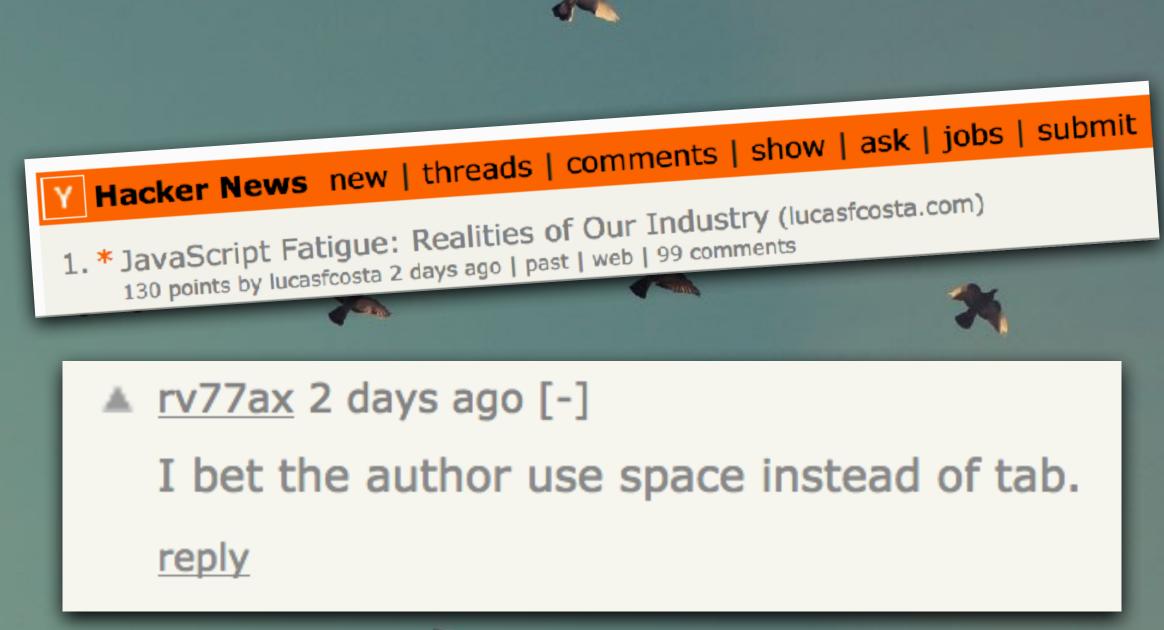


LONDON









# Right, but will you talk about bird?

# This is a talk about Lambda Calculus

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# Calculus III Numbers









# Lambda Calculus s what s behind Functiona Programming

# Is it useful?



#### How helpful is knowing lambda calculus? [closed]

To all the people who know <u>lambda calculus</u>: What benefit has it bought you, regarding programming? Would you recommend that people learn it?

69

math

functional-programming computer-science lambda-calculus

\_\_\_\_ 10C

17

26

If you want to program in any <u>functional programming language</u>, it's essential. I mean, how useful is it to know about Turing machines? Well, if you write C, the language paradigm is quite close to Turing machines -- you have an instruction pointer and a current instruction, and the machine takes some action in the current state, and then ambles along to the next instruction.

In a functional language, you simply can't think like that -- that's not the language paradigm. You have to think back to lambda calculus, and how terms are evaluated there. It will be much harder for you to be effective in a functional language if you don't know lambda calculus.

share improve this answer

answered Sep 22 '08 at 12:55



#### If you want to program in any functional programming language, it's essential.

# essential.

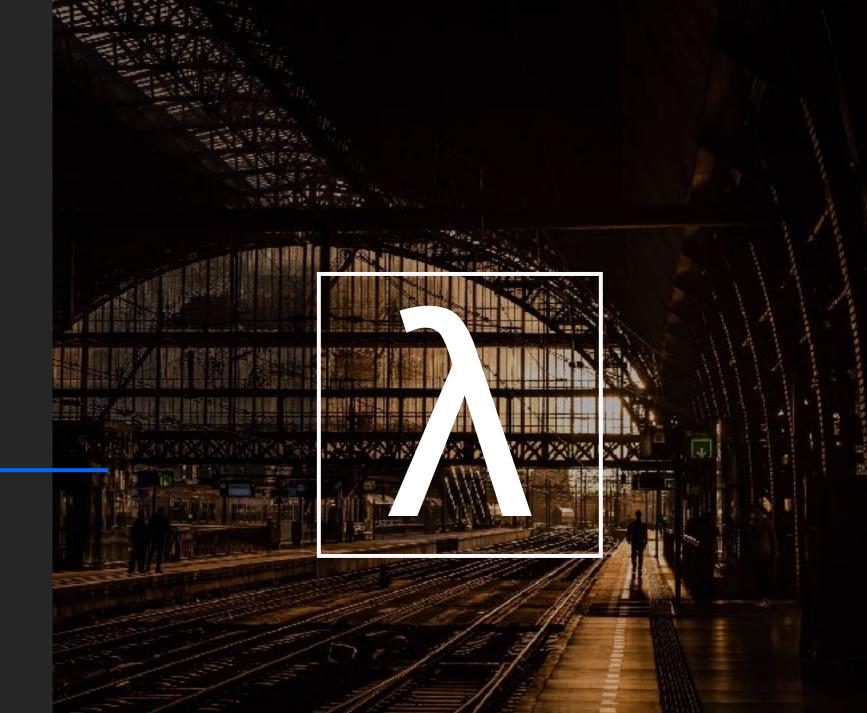


### In Lambda Calculus Everything is an expression

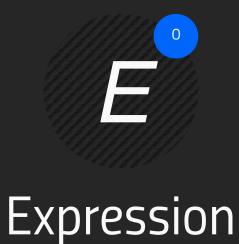
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### In Lambda Calculus Everything evaluates to a value



# This is Lambda Calculus





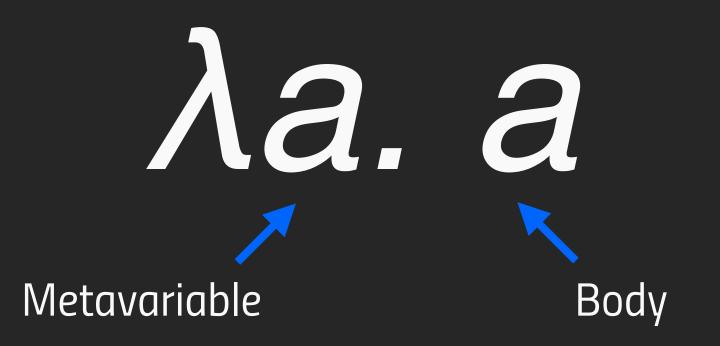
### RUINING JAVASCRIPT This is Lambda Calculus



#### RUINING JAVASCRIPT This is a lambda

# Ja. a

#### ruining javascript This is a lambda



#### RUINING JAVASCRIPT This is a lambda

a(arg)

# Applying arg to a

# RUINING JAVASCRIPT

*λa*. *a* 

and the set back arg

#### RUINING JAVASCRIPT This is a lambda

# $\lambda a. a$

## It can only take a single argument!

#### ruining javascript This is a lambda

# **Na.Nb.b**Lambdas can return other lambdas

#### ruining javascript This is a lambda

# λa. λb. b Currying

# RUINING JAVASCRIPT

# $a \Rightarrow b \Rightarrow b$

### In JavaScript...

#### RUINING JAVASCRIPT This is a lambda

### •••

const add = (a) => (b) => a + b; const addTwo = add(2);

console.log(addTwo(3)); // Five

# This is a lambda

#### DivideThreeNumbers :: Int -> Int -> Int -> Int

### In Haskell...

#### ruining javascript This is a lambda

# $(\lambda x. x b) [x \rightarrow y]$

evaluation or substitution

# *(y b)*

#### evaluation or substitution

# $(\lambda x. x b) [b \rightarrow y]$

### RUINING JAVASCRIPT

# $(\lambda x. x b) [b \rightarrow y]$

b is not an argument!

# Each evaluation step is called a beta-reduction

### RUINING JAVASCRIPT

We can only do beta reduction when expressions have beta-reduxes

We can only do beta reduction when expressions contain an applicative form

### When an expression cannot be further evaluated it is said to be in its beta-normal form

### $\lambda x. \lambda y. + y (+ x 1)$

### RUINING JAVASCRIPT

### $\lambda x. \lambda y. + y (+ x 1) 4 2$

# $(\lambda x. \lambda y. + y (+ x 1)) 4 2$ $(\lambda x. \lambda y. + y (+ 4 1)) [x \rightarrow 4]$

# $(\lambda x. \lambda y. + y (+ x 1)) 4 2$ $(\lambda x. \lambda y. + y (+ 4 1)) [x \rightarrow 4]$ $(\lambda y. + y (+ 4 1))$

### (λy. + y (+ 4 1)) 2

# $(\lambda y. + y (+ 4 1)) 2$ $(\lambda y. + y (+ 4 1)) [y \rightarrow 2]$

 $(\lambda y. + y (+ 4 1)) 2$  $(\lambda y. + y (+ 4 1)) [y \rightarrow 2]$  $(+ 2 (+ 4 1)) [y \rightarrow 2]$ 

# $(\lambda y. + y(+ 4 1)) 2$ $(\lambda y. + y(+ 4 1))[y \rightarrow 2]$ (+ 2 (+ 4 1)) [y→2] (+2(+41))

### (+ 2 (+ 4 1)) (+ 2 5) (7)

# XJZ

# XJZ

#### How do we disambiguate?

# $\frac{X(YZ)}{2} \text{ or } (XY)Z$

# (X Y) Z applications are left associative

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100

# fn(first)(second)

just like when you write proper functional code

# 

How do we disambiguate?

# $(\lambda x. x) y_{or} (\lambda x. x y)$

# 

abstractions extend as much to the far-right as possible

### RUINING JAVASCRIPT

# this is why you might need parenthesis to disambiguate

### Lambda Ca Cu us is another way of representing computation

# Lambda Calculus is Turing Complete

### le can replace any programming language with Lambda Calculus

# gonna write Java Script with nothing but functions

#### I'm not the first one.

#### PROGRAMMING WITH NOTHING

@tomstuart - RUSY MANOR - 2011-10-29

#### Programming with nothing

By Tom Stuart

#### I'm not the first one. But I want to take it one step further.



@tomstuart - RUSY MANOR - 2011-10-29

#### Programming with nothing

By Tom Stuart





Part 1



Part 2



Part 3



Part 1



Part 2



Part 3



Part 4



# With nothing but functions

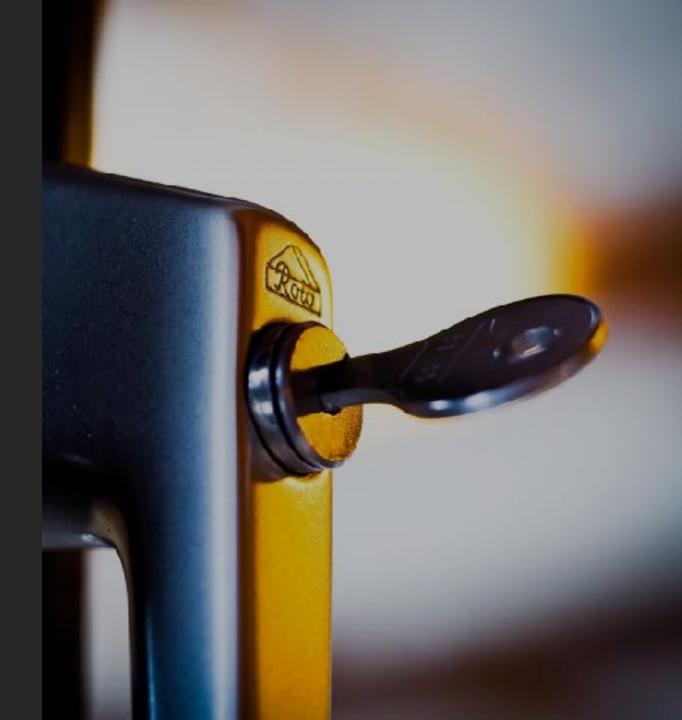
- •All functions must take only one argument
- Functions can return other functions
- •We will use assignments to make things easier to explain

## Ruining JavaScript

Part 1

73

#### Replacing Booleans



Part 2



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Representation





Meaning



#### How can we represent quantities with functions?

#### How can we represent quantities with functions? Function applications!



#### const ZER0 = f => x => x

How can we represent quantities with functions? Function applications!

#### •••

const ZERO = f => x => x const ONE = f => x => f(x)

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How can we represent quantities with functions? Function applications!

const ZER0 = f => x => x const ONE = f => x => f(x) const TW0 = f => x => f(f(x))

#### How can we represent quantities with functions? Function applications!

#### •••

const ZER0 = f => x => x const ONE = f => x => f(x) const TW0 = f => x => f(f(x)) const THREE = f => x => f(f(f(x))) const FOUR = f => x => f(f(f(f(x)))) const FUVE = f => x => f(f(f(f(f(x))))) const FIVE = f => x => f(f(f(f(f(f(x)))))))

#### Part 3

## Replacing Arithmetics

Integer) swith 2 systement 1-

C Rys 2) Histogramy przedstawnają z polskich but w ramach 2, sebrowanych EPSTAL u śroc a) R<sub>a</sub> – granica plastyczności (m b) R<sub>a</sub> – wytrzymałość na rozcia c) A<sub>gr</sub> – wydłużenie przy maksymi

PN-EN 10080-2005), ktory wymaga 1 Wyteme badania wy od producents prowadrenią wewnętrz- | cję zakładu, ciągły n ney kontrols produkcys, wykonywania

wyróżnia system 1 hadania soutatwork w rakindaw.

ю











Pairs

# const WRAP = (a) => (f) => f(a)

Pairs

## const WRAP = (a) => (f) => f(a)

## First we store something.

Pairs

# const WRAP = (a) => (f) => f(a)

#### Then we apply whatever is stored to a function



Pairs

#### const PAIR = $x \Rightarrow y \Rightarrow f \Rightarrow f(x)(y)$

Pairs

#### const PAIR = x = y = f = f(x)(y)

#### First we store two values.

Pairs

#### const PAIR = $x \Rightarrow y \Rightarrow (f) \Rightarrow f(x)(y)$

#### Then we tell which one we want.

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Pairs

const TRUE =  $a \Rightarrow b \Rightarrow a$ const FALSE =  $a \Rightarrow b \Rightarrow b$ 

const PAIR =  $x \Rightarrow y \Rightarrow f \Rightarrow f(x)(y)$ 

Pairs

const FIRST =  $a \Rightarrow b \Rightarrow a$ const SECOND =  $a \Rightarrow b \Rightarrow b$ 

const PAIR =  $x \Rightarrow y \Rightarrow f \Rightarrow f(x)(y)$ 



#### Incrementing Pairs

## $\{0,0\}$ $\{0,1\}$

#### **Incrementing Pairs**

# {0,0} {0,1} {1,2}

#### Incrementing Pairs

## $\{0,0\}$ $\{0,1\}$ $\{1,2\}$ $\{2,3\}$

#### Incrementing Pairs

## $\{0,0\}$ $\{0,1\}$ $\{1,2\}$ $\{2,3\}$ $\{4,5\}$

#### **Incrementing Pairs**

## $\{0,0\}$ $\{0,1\}$ $\{1,2\}$ $\{2,3\}$ $\{4,5\}$ $\{5,6\}$

#### **Incrementing Pairs**

## $\{0,0\}$ $\{0,1\}$ $\{1, 2\}$ $\{2, 3\}$ {4,5} {5,6} Five $\{0, 0\}$

#### **Incrementing Pairs**

## $\{0,0\}$ $\{0,1\}$ $\{1, 2\}$ $\{2, 3\}$ {4,5} {5,6} Five $\{0, 0\}$ $\{0, 1\}_{x_1}$

#### **Incrementing Pairs**

## $\{0, 0\} \{0, 1\}$ $\{1, 2\}$ $\{2, 3\}$ {4,5} {5,6} Five $\{0, 0\}$ $\{1, 2\}_{x^2}$

Replacing arithmetics

### **Incrementing Pairs**

# $\{0, 0\} \{0, 1\}$ $\{1, 2\}$ $\{2, 3\}$ {4,5} {5,6} Five $\{0, 0\}$ $\{2, 3\}_{x3}$

Replacing arithmetics

### **Incrementing Pairs**

# $\{0, 0\} \{0, 1\}$ $\{1, 2\}$ $\{2, 3\}$ {4,5} {5,6} Five $\{0, 0\}$ $\{3, 4\}_{x4}$

Replacing arithmetics

### Incrementing Pairs

# $\{0, 0\} \{0, 1\}$ $\{1, 2\}$ $\{2, 3\}$ $\{4, 5\}$ $\{5, 6\}$ {4,5} Five $\{0,0\}$

Predecessor



Replacing arithmetics The Predecessor Function

#### •••

#### const PREDECESSOR = n => n(INCREMENT\_PAIR)(PAIR(ZER0)(ZER0))(TRUE)

# First of N x INCREMENT\_PAIR(0, 0)

#### Replacing arithmetics The Subtraction Function

- - -

#### const SUBTRACTION = $n \Rightarrow k \Rightarrow k(PREDECESSOR)(n)$

## Predecessor of N K times

Replacing Boolean Operators



### RUINING JAVASCRIPT Alpha Equivalence



### const FALSE = $a \Rightarrow b \Rightarrow b$ const ZERO = $f \Rightarrow x \Rightarrow x$

# $\lambda x. \lambda y. x(y(z)) \{a/x\}$

Renaming "x" to "a"

# $\lambda a. \lambda y. a(y(z))$

# $\lambda a. \lambda y. a(y(z)) \{b/y\}$

Renaming "y" to "b"

# $\lambda a. \lambda b. a(b(z))$

# $\lambda a. \lambda y. a(y(z)) \{c/z\}$

Renaming "z" to "c"

# $\lambda a. \lambda y. a(y(z))$ Same thing! Z is not an argument!

# Ruining JavaScript With Birds

Part 2

## Combinators

Functions that don't have free variables.

const withContext = a => b(a)
 a is bound and b is free

const combinator = a => b => a
 both a and b are bound

## Combinators

### 

#### const one = $x \Rightarrow x - x$

## Combinators

### •••

const two =  $x \Rightarrow x + y$ 

## Combinators

### •••

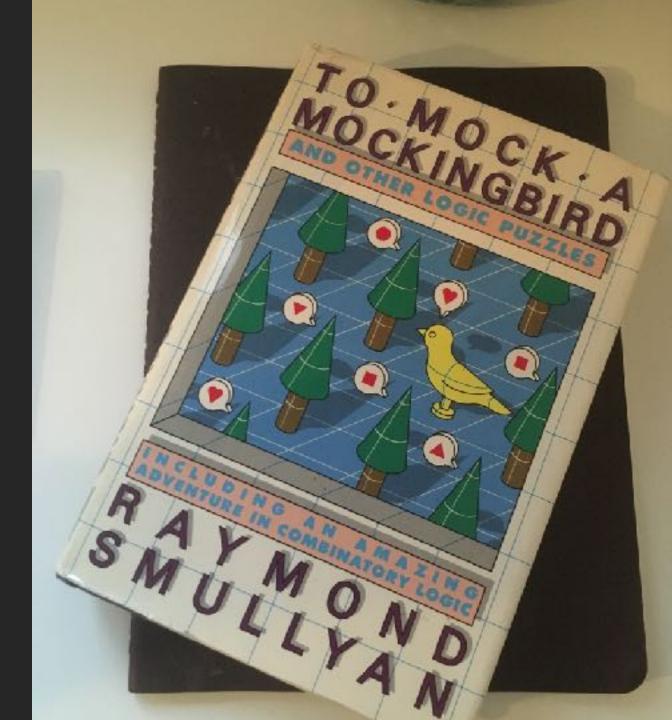
### const three = $x \Rightarrow y + z$

## Combinators

### 

### const four = $(x) \Rightarrow (y) \Rightarrow x + y$

### Combinators and Birds To Mocka Mockingbird



## "Combinatory logic is a notation to eliminate the need for quantified variables in mathematical logic."

Combinators
The Idiot Bird

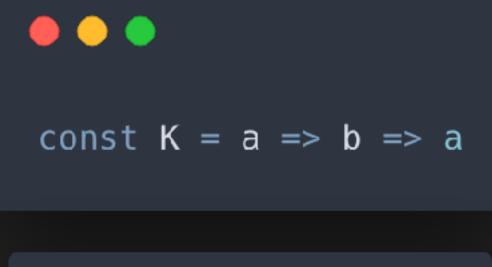
### 

### const i = $x \Rightarrow x$

### 

### const IF = x => x

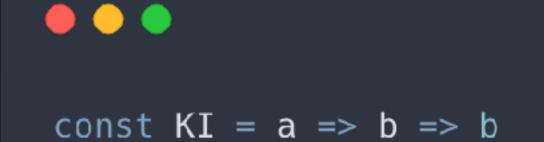
### Combinators The Kestrel



•••

const TRUE =  $a \implies b \implies a$ 

### Combinators The Kite



•••

const FALSE = a => b => b

Combinators
The Cardinal

# const C = f => a => b => f(b)(a)

•••

const NOT = f => a => b => f(b)(a)

### Combinators The Vireo

## const V = f => a => b => f(a)(b)

### •••

const PAIR =  $f \Rightarrow a \Rightarrow b \Rightarrow f(a)(b)$ 

Combinators The Bluebird

.....

### const $B = f \Rightarrow g \Rightarrow a \Rightarrow f(g(a))$

.....

const MULTIPLICATION =  $n \Rightarrow k \Rightarrow f \Rightarrow n(k(f))$ 

### Combinators The Thrush

## const Th = a => f => f(a)

## Combinators The Starling



### const S = f => g => x => f(x)(g(x))

# One last combinator



#### Why did you choose the name "Y Combinator?"

The Y combinator is one of the coolest ideas in computer science. It's also a metaphor for what we do. It's a program that runs programs; we're a company that helps start companies.

### Y: The Most Beautiful Idea in Computer Science explained in JavaScript

20th of May, 2018 — Lucas Fernandes da Costa at London, United Kingdom 🗱

### bit.ly/2PLFJkn - lucasfcosta.com

### Replacing Functions (Successor)

## <u>const B = f => g => a => f(g(a))</u>

### const SUCCESSOR = n => f => x => f(n(f)(x))

#### Combinators Replacing Functions (Successor)

### const $B = f \Rightarrow g \Rightarrow a \Rightarrow f(g(a))$

### const SUCCESSOR = n => f => B(f)(n(f))

#### Combinators Replacing Functions (Successor)

#### const S = f => g => x => f(x)(g(x))

#### const SUCCESSOR = n => f => B(f)(n(f))

#### Combinators Replacing Functions (Successor)

#### const S = f => g => x => f(x)(g(x))

const SUCCESSOR = n => f => B(f)(n(f))
 const SUCCESSOR = S(B)

Combinators Replacing Functions (Addition)

- - -

const SUCCESSOR = S(B)

### const ADDITION = n => k => n(SUCESSOR)(k)

Combinators Replacing Functions (Addition)

- - -

const SUCCESSOR = S(B)

### const ADDITION = n => k => n(S(B))(k)

### Replacing Functions (Addition)

const Th = a => f => f(a)

### const ADDITION = k => Th(S(B))(k)

### Replacing Functions (Addition)

const C = f => a => b => f(b)(a)

const Th = a => f => f(a)

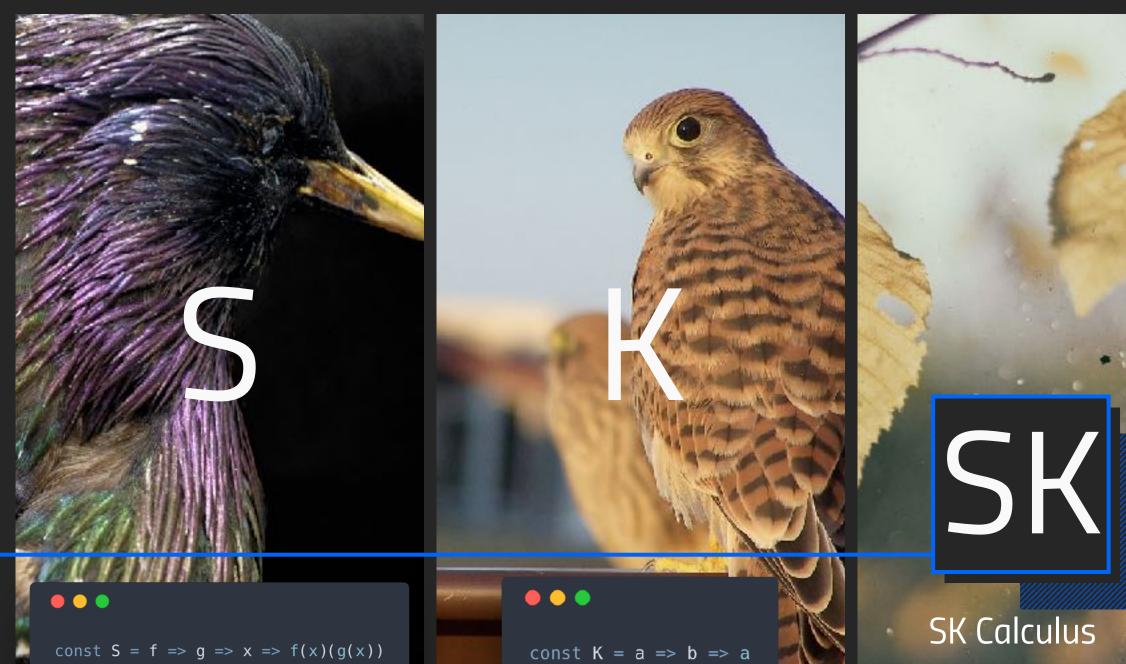
### const ADDITION = C(Th)(Th(S(B)))

## Ruining JavaScript and the Birds

Part 3

## WHAT IF I TOLD YOU YOU ONLY NEED TO CONSTRUCTORS?

Yes, that's right. Two.



### People also call it SKI Calculus

## People also call it SKI Calculus Because it's more <u>convenient</u> to have I

## People also call it SKI Calculus Because it's more <u>convenient</u> to have I const I = S(K)(K)

## const KI = K(S(K)(K))

## const KI = K(I)

## const KI = K(I)const B = S(K(S))(K)

## const KI = K(I)const B = S(K(S))(K)const C = Of course!

#### SK Calculus

#### http://www.angelfire.com/tx4/cus/combinator/birds.html

Combinator Birds				
Function Abstraction	Symbol	Diel	Combinator	3K Combinator
labc.a(bc)	в	Bluetind	30X3)K	COURSES
labed a(bed)	8 <sub>1</sub>	Blackbird	BBB	(\$(K((\$K\$))K)))(\$(K\$)K))
labede.a(brde)	B2	Bunting	B(B3B)B	(\$(K((\$K((\$(K()(K))))(\$(K\$)(K))))((\$(K\$)(K))
labed a(b(ed))	B3	Becard	B(B3)E	(36K(5)K(3(K5)K))(5(K5)K)))(5(K5)K))
labe.aeb	с	Cardinal	5(3B8)(EK)	(\$(\$(\$(K)(\$(\$K\$))K))(\$)(KK))
labed ab(od)	D	Dave	BB	(3(K()3(K3)K3))
labede.abc/de)	D1	Dickdesel	D(B3)	306(5)(63)(50)(0)
labcds.a(5c)(ds)	D2	Devekier	BB(3B)	(\$\$K((\$K\$),K)))(\$K((\$(K\$))K)))
labcde.ab(ede)	E	Eagle	D(B3B)	(S(K)(S(KS))K))((S(K3))K)))
labcdefg.alboc3(e/g)	È	Bald Eagle	B(B3B(B(BBB))	(SIKUS KUSUKUSUKUSUKUSUKUSUKUSUKUSUKUSUKUSU
labc.cba	F	Fisch	ETTET	((5(K)(5)(5K,K))(K)(5(K)(5(K)))(K))))(((5)K)(5(K)(5(
labed ad(se)	G	Geldfinch	BBC	(5(K)(5)(5)(5)(5((5(K)(5(K5)(3))))(K)))
labc.abcb	Ħ	Haramingbiad	5W(BC)	(3(K)(3(K)(3(K)(3)(3K)K)))(3(K)(5)))(3(K)(3(K
ha		Identity Bird (aka Idoot)	SKK	(SKIK)
labed ab(ade)	-	iny.	B(BC)(W(BC(B(BEB))))	(SK(S(K)(S(K)(S(K))K))(S(K)))(S(S(K)(S(SK)K))(SK)K))(S(K)S(K)
lab.a	ĸ	Kestrel (True)	K	x
(dd)a.dd	L	Lark	CBM	(5((5(K))K)(K(S)(SKK))((SK)K)))
haa	м	Moelkingbird	SE	(\$((\$K)K))((\$K,K))
lab.ab(ab)	M2	Double Mockinghiad	DM	(3(K)(5(5K)K))(3(5K)K)))
lab.b(ab)	0	Ow	51	(30(854)8.0)
labe.b(ac)	Q	Queer Bird	CB	(\$(K(\$)(\$(K\$)(K)))(K)
labc.s(cb)	Q1	Quisotic Bird	DCE	(\$\$K(\$(\$(\$(K5)(K)))\$)((KK)))((3(K5)K))
lahc.h(ca)	Q2	Quinzical Bird	C(BCB)	(NKG05(K)(305(K)(3050)) N (0005(K30K)00K)
labc.c(sb)	Q,	Quirky Bind	BT	(S(K)(5(K(S(K)K))))K))
labc.c(ba)	Q.4	Quacky Bird	F*B	CORES CONSERVENTS DEREMINATED STREET CONSTRUCTION OF A DESCRIPTION OF A DE
labe.bea	R	Robin	BBT	(\$\$K\$;K\$);K);0(\$(K;\$(\$K)K);0);K)+
labc.ac(bx)	5	Studing	5	3
lab.ba	T	Thrush	CI	(3(K(3)(2K)K)))K)
lab b(ash)	U	Turing	1.0	(SIKGSOSKIKDOI(SCSKIKOI(SKJKO)
labc.cab	v	Virso (aka Pairing)	рст	(\$4K(65(\$4K(5(K5)(K)))\$)(KK0))((\$4K(\$(\$K)K))(K))

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If we can replace all code by functions

If we can replace all code by functions Replace all functions by combinators

If we can replace all code by functions Replace all functions by combinators And replace all combinators by S and K

If we can replace all code by functions Replace all functions by combinators And replace all combinators by S and K

Then we can replace all code by S and K

https://crypto.stanford.edu/~blynn/lambda/sk.html http://xn--wxak1a.com/blog/Combinators.html

## Apologising

Part 4

### Functional Programming Computability Theory Mathsm Apologising

Combinatory Logic

Recursion

Wanting to frame Gödel pictures to hang in your room

**Compiler Theory** 

Part 4

# References

http://codon.com/programming-with-nothing The blog post for the talk I mentioned in the beginning https://speakerdeck.com/tomstuart/programming-with-nothing Slides for the talk "Programming with Nothing" http://www.angelfire.com/tx4/cus/combinator/birds.html List of Notorious Combinators https://bit.ly/2xpcPKn A Flock of Functions - Gabriel Lebec https://amzn.to/2BVVsa1 To Mock a Mockingbird - Raymond Smullyan http://computationbook.com Understanding Computation - Tom Stuart https://www.youtube.com/watch?v=\_kYGDJSm0gE - ASU Lectures - Lambda Calculus by Adam Doupé

# Thank you!

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@LUCASFCOSTA (GITHUB)
LUCASFCOSTA.COM

Shhh, no tears. Only lambdas now.